6.1 Introduction to Similarity

Objective: Students will be able to identify similar polygons. Students will solve for missing sides of a similar polygons. Students will determine if triangles are similar using theorems.

Warm – Up

Solve the proportion.

1. $\frac{y}{20} = \frac{3}{10}$

$\frac{10y}{10} = \frac{60}{10}$

$\boxed{y = 6}$

2. $\frac{4}{a-3} = \frac{2}{5}$

$2(a-3) = 20$

$2a - 6 = 20$

$2a = 26$

$a = 13$

3. $\frac{2x+5}{3} = \frac{x-5}{4}$

$4(2x+5) = 3(x-5)$

$8x + 20 = 3x - 15$

$5x = -35$

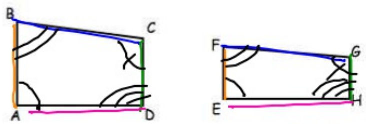
$x = -7$

Similar Polygons *Same shape but different sizes

Two polygons are **similar** polygons if : *SIDES are proportional.

$\sim$ = similiar *ANGLES are congruent.

Statement of proportionality



Ratios of corresponding sides

$\frac{AB}{EF} = \frac{BC}{FG} = \frac{CD}{GH} = \frac{DA}{HE}$

Corresponding angles

$\angle A \cong \angle E$, $\angle B \cong \angle F$, $\angle C \cong \angle G$ and $\angle D \cong \angle H$

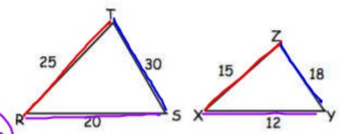
Similarity Statement: ABCD ~ EFGH

Examples:

1. Use the diagram, ΔRST ~ ΔXYZ (similar)
   a. List all pairs of congruent angles.

   b. Check that the ratios of corresponding side lengths are equal.

   $\frac{25}{15} = \frac{5}{3}$ $\frac{30}{18} = \frac{5}{3}$ $\frac{20}{12} = \frac{5}{3}$



   c. Write the ratios of the corresponding side lengths in a ***statement of proportionality***.

   $\frac{RT}{XZ} = \frac{TS}{ZY} = \frac{RS}{XY}$

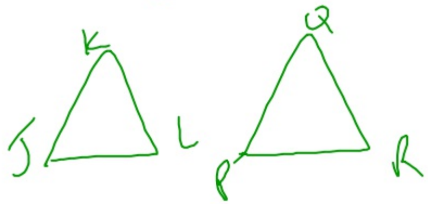
2. Given ΔJKL ~ ΔPQR, list all the pairs of congruent angles. Write the ratios of the corresponding side lengths in a statement of proportionality.

$\angle J \cong \angle P$

$\angle K \cong \angle Q$

$\angle L \cong \angle R$

$\frac{JK}{PQ} = \frac{KL}{QR} = \frac{JL}{PR}$



If two polygons are similar, then the ratio of the lengths of two corresponding sides is called a scale factor.

In number one, the polygons have common ratio of $\frac{5}{3}$ which is the scale factor of ΔRST ~ ΔXYZ.

4:5

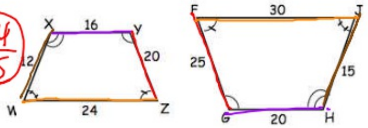
3. Determine whether the polygons are similar. If they are, write a similarity statement and find the scale factor of ZYXW to FGHJ.

① ②

Yes, XYZW ~ HGFJ, S.F. = $\frac{4}{5}$

$\frac{12}{15} = \frac{4}{5}$ $\frac{16}{20} = \frac{4}{5}$ $\frac{20}{25} = \frac{4}{5}$

$\frac{24}{30} = \frac{4}{5}$

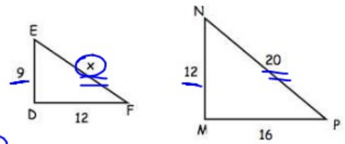


4. In the diagram, ΔDEF ~ ΔMNP. Find the value of x.

$\frac{9}{12} = \frac{x}{20}$

$\frac{12x}{12} = \frac{180}{12}$

$x = 15$



5. In the diagram, ABCD ~ QRST

a. What is the scale factor of QRST to ABCD?

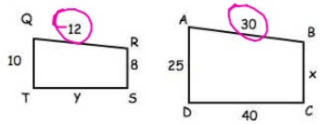
$\frac{12}{30} = \frac{2}{5}$

$\frac{8}{x} = \frac{2}{5}$ $\frac{2x}{2} = \frac{40}{2}$ $x = 20$

b. Find the value of x.

$\frac{8}{x} = \frac{10}{25}$ $\frac{10x}{10} = \frac{200}{10}$

$x = 20$



c. Find the value of y.

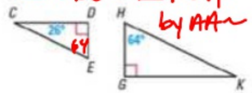
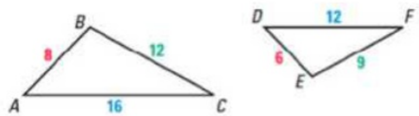
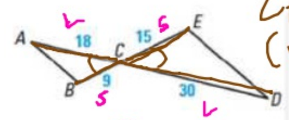
$\frac{y}{40} = \frac{2}{5}$

$\frac{5y}{5} = \frac{80}{5}$

$y = 16$

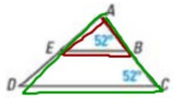
6.2 PROVING TRIANGLES SIMILAR

There are three different ways to prove triangles similar: AA, SSS, SAS.

Angle-Angle Similarity (AA) • Two corresponding angles must be CONGRUENT.	Show the triangles are similar and explain your reasoning and write a similarity statement. $\angle D \cong \angle G$ $\angle E \cong \angle H$ a.  If $\angle K \cong \angle Y$ & $\angle J \cong \angle X$ then $\triangle JKL \sim \triangle XYZ$ by AA~ b. $\triangle CDE \sim \triangle KGH$ by AA~ 
Side-Side-Side Similarity (SSS) * All sides must be PROPORTIONAL. * $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$	Show the triangles are similar and explain your reasoning and write a similarity statement.  $\frac{8}{6} = \frac{4}{3}$ $\frac{12}{9} = \frac{4}{3}$ $\frac{16}{12} = \frac{4}{3}$ $\triangle ABC \sim \triangle DEF$ by SSS~
Side-Angle-Side Similarity (SAS) * 2 sides must be PROPORTIONAL. * Included Angle must be CONGRUENT. **Included angle is in between the two sides	Show the triangles are similar and explain your reasoning and write a similarity statement.  $\angle ACB \cong \angle DCE$ (vertical $\angle$'s) $\frac{9}{15} = \frac{3}{5}$ $\frac{18}{30} = \frac{3}{5}$ ✓ $\triangle BAC \sim \triangle EDC$ by SAS~

Explain how you know the triangles are similar.

a. $\triangle ABE$ and $\triangle ACD$

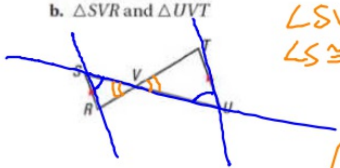


$\angle B \cong \angle C$

$\angle A \cong \angle A$ (reflexive property)

yes by AA~

b. $\triangle SVR$ and $\triangle UVT$

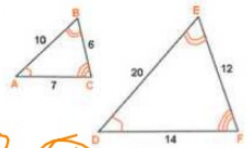


$\angle SVR \cong \angle UVT$ (vertical

$\angle S \cong \angle U$ (alt. int. $\angle$s)

yes by AA~

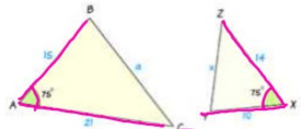
c.



$\frac{10}{20} = \frac{1}{2}$ $\frac{6}{12} = \frac{1}{2}$ $\frac{7}{14} = \frac{1}{2}$

yes by SSS~

d.



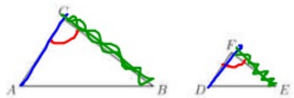
$\frac{15}{10} = \frac{3}{2}$

$\angle A \cong \angle X$

$\frac{21}{14} = \frac{3}{2}$

yes by SAS~

Use the following diagram for e-f



e. Given $\angle C \cong \angle F$, what additional information would you need to show the triangles are similar by AA?

$\angle A \cong \angle D$ OR $\angle B \cong \angle E$

f. Given $\overline{AC}$ is proportional to $\overline{DF}$ and $\angle C \cong \angle F$ What additional information would you need to show the triangles are similar by SAS?

$\frac{\overline{CB}}{\overline{FE}}$