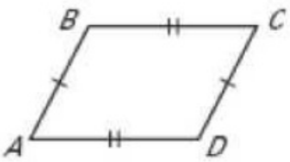
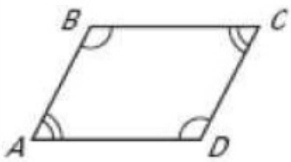
8.3 Show that a Quadrilateral is a Parallelogram

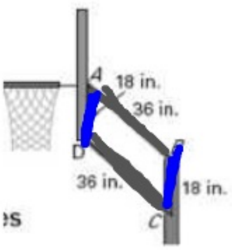
Goal • Use properties to identify parallelograms.

A QUADRILATERAL IS A parallelogram IF ....

Theorem 8.7 It's opposite sides are ≅	 If $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{AD}$, then *ABCD* is a parallelogram.
Theorem 8.8 Its opposite ∠'s are ≅.	 If $\angle A \cong \angle C$ and $\angle B \cong \angle D$, then *ABCD* is a parallelogram.

EXAMPLE 1:

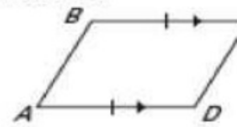
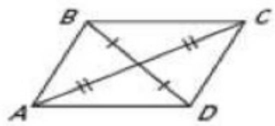
Basketball In the diagram at the right, $\overline{AB}$ and $\overline{DC}$ represent adjustable supports of a basketball hoop. *Explain* why $\overline{AD}$ is always parallel to $\overline{BC}$.



ABCD is a ▱ since its opposite sides are ≅

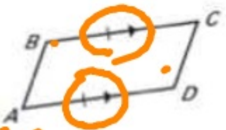
∴ opposite sides are ∥ making $\overline{AD} \parallel \overline{BC}$ always.

A QUADRILATERAL IS A parallelogram IF ....

Theorem 8.9 One pair of opposite sides are $\cong$ + $\parallel$.	 If $\overline{BC} \parallel \overline{AD}$ + $\overline{BC} \cong \overline{AD}$ Then ABCD is a ▱.
Theorem 8.10 Diagonals bisect	

EXAMPLE 2:

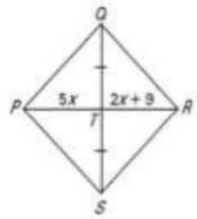
Lights The headlights of a car have the shape shown at the right. *Explain* how you know that $\angle B \cong \angle D$.



ABCD is a ▱ because one pair opp sides are $\cong$ + $\parallel$.

$\therefore$ $\angle B \cong \angle D$ because opp $\angle$'s are $\cong$ in a ▱.

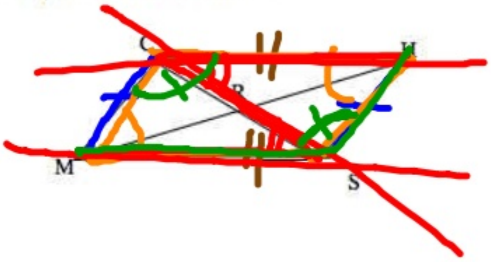
EXAMPLE 3: For what value of x is PQRS a parallelogram?



$$5x = 2x + 9$$
$$-2x \quad -2x$$
$$\frac{3x}{3} = \frac{9}{3}$$
$$\boxed{x = 3}$$

YOU TRY!

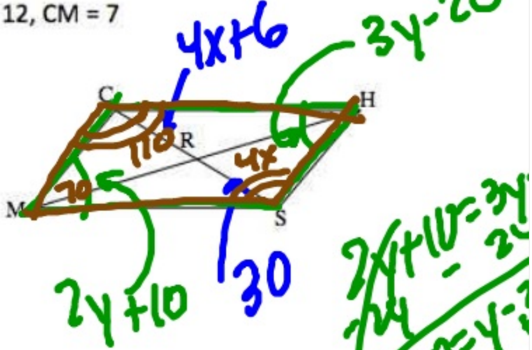
Use the diagram of parallelogram MCHS to complete each statement.



1. $\overline{CH} \cong$ $\overline{MS}$
2. $\overline{MC} \cong$ $\overline{SH}$
3. $\angle CHS \cong$ $\angle SMC$
4. $\angle HCS \cong$ $\angle MSC$
5. $\angle HSM \cong$ $\angle HCM$

Use the diagram of parallelogram MCHS and the given information to find the indicated measures.

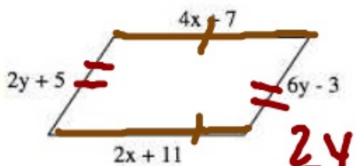
GIVEN: m∠CMS = 60°, m∠HCS = 22°, CH = 8, RS = 4, MH = 12, CM = 7



6. m∠MCH = 120°
7. CS = 8
8. RH = 6
9. m∠MSH= 120°
10. m∠CSM= 22°
11. MS = 8

Find the value of each variable in the parallelogram.

12.



$4x+7=2x+11$

$2x+7=11$

$2x=4$

$2y+5=6y-3$

$5=4y-3$

$8=4y$

x = 2 y = 2

13. In the diagram above, assume that CR= 4x+6 and RS= 30. Find the value of x.

$4x+6=30$

$4x=24$

$\boxed{x=6}$

$2y+10=3y-20$

$10=y-20$

14. In the diagram above, assume that:

m∠CHS= 3y-20 and
m∠CMS = 2y +10.

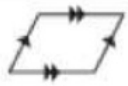
(a) Find the value of y:
30

(b) Find m∠MCH
110

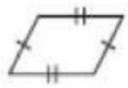
(c) If m∠HSM=4x, find x.
$4x=110$
$\boxed{x=27.5}$

CONCEPT SUMMARY: WAYS TO PROVE A QUADRILATERAL IS A PARALLELOGRAM

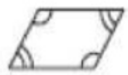
1. Show both pairs of opposite sides are parallel. **(Definition)**



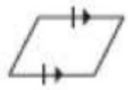
2. Show both pairs of opposite sides are congruent. **(Theorem 8.7)**



3. Show both pairs of opposite angles are congruent. **(Theorem 8.8)**



4. Show one pair of opposite sides are congruent and parallel. **(Theorem 8.9)**



5. Show the diagonals bisect each other. **(Theorem 8.10)**